World Transactions on Engineering and Technology Education
Vol.9, No.3, 2011
© 2011 WIETE

Assessing the modelling competencies of engineering students

Chih-Hsien Huang

Ming Chi University of Technology
New Taipei City, Taiwan

ABSTRACT: This research is based on models and modelling perspectives. The subjects were 58 university of technology freshman engineering students. The research tools included six mathematical modelling activities, mathematical modelling tests and semi-structured interviews. In this article, modelling competencies and their development are being dealt with. These competencies are evaluated by means of a t est that was applied to all participating students at the beginning and at the end of the mathematical modelling course. The results of these tests are reported and they confirm that mathematical modelling competencies can be improved through participation in an appropriate mathematical modelling course.

INTRODUCTION

Concerned with the negative opinions held by students towards mathematics, more and more mathematics researchers posit that applying real world problems to the classroom is a good way to link the mathematical world to the real world. Many researchers in engineering education agree that mathematical modelling is an important aspect of engineering education [1-4]. For the past several decades, applied mathematics has been playing an increasingly important role in other disciplines such as engineering, nanotechnology, economics and biology. Many mathematics educators and engineering education researchers believe that this practice should also be extended to classrooms through modelling activities, by offering additional tools for students to use outside school mathematics, exposing them to the *real life* mathematics outside the classroom. Therefore, to keep up with the new technological era, it is essential to cultivate students' mathematical literacy by promoting mathematics instruction from a modelling perspective.

The purpose of mathematical modelling is to support the learning of mathematics for engineering students [5]. Through modelling, mathematics can be used to describe, understand and predict the perceived reality of our real lives. Therefore, mathematical modelling can help students reinforce the link between their mathematical experiences outside school and the mathematical problems posed in the modelling activities. Mathematical modelling involves a wide variety of procedures, such as mathematisation, interpreting and communication, and even the processes of its application [3][6]. Unlike conventional problem-solving, which concentrates only on mathematical representation and results, mathematical modelling places the focus upon transforming and interpreting situational information, identifying potential problems, developing a pattern, and re-interpreting the premises, hypothesis, and possible deviations of mathematical answers. These procedures usually proceed in stages, through which the students continuously develop and refine their mathematical models. In the modelling process just described, the students must acquire the capabilities to engage themselves in mental activities for stage transition and transcendence.

Calculus is a fundamental course in university mathematics education, and students must attain an understanding of the concepts related to calculus, and should be able to put them into practice. For engineering students, calculus is even more essential to their professional coursework and future careers. To take the acquisition of conceptual knowledge, procedural knowledge and mathematical modelling competence into account, the *island approach* is adopted, incorporating modelling activities into formal activities for calculus instruction, so as to avoid the impedance of the first year students who are accustomed to the traditional way of instruction; this is a feasible approach for instruction [7]. Therefore, this study aims to design six mathematical modelling activities to be embedded into the calculus curriculum from a pattern-based and modelling-oriented perspective. Through instructional intervention, the mathematical modelling competence of first year engineering students is gradually developed, and their progress is evaluated through mathematical modelling competence tests.

This study takes on a graphical representation of the modelling process to capture and define the elements pertaining to modelling competence [3]. This sets the context in line with the definition of mathematical modelling competence by Blomhøj and Jensen: *By mathematical modelling competence, we mean the ability to autonomously and insightfully carry out all aspects of the mathematical modelling process in a certain context* [8]. Mathematical modelling competence refers to the capabilities required to complete a mathematical model. This includes: 1) formulation of a task by identifying the characteristics of the perceived reality - in other words, putting oneself into the context to construct a real life model for the situation; 2) selection of the relevant objects and relationships to make a mathematical representation possible; 3) translation of these objects and relationships from their initial mode of appearance into mathematics; 4) use of mathematical methods to achieve mathematical results and conclusions; 5) interpretation of these results and conclusions regarding the initial domain of inquiry; 6) evaluation of the validity of the model through comparison with observed or predicted data, or with theoretically based knowledge; 7) review of the mathematical models against the expected results for the initial domain of inquiry, going back to the modelling process, if the results conform to expectations.

Many studies stress a particular stage of the modelling process; these questions are posed to students to test their capability to transition from perceived reality to the mathematical world, and can determine the difficulties that students are facing, and the extent to which the concepts are understood [9-11]. The mathematical modelling test in this study adopts choice-based questions, consisting of 22 multiple choice questions in eight categories, with each category testing a sub-competence of modelling [12-14]. Each category has a different context, and a diversified context keeps the students' performance in modelling from being affected by familiarity with a particular context. Each category contains at least two questions that are designed for situational matching. Each question has five options, including one correct option, and four incorrect, but deceptive ones. This study selects 14 out of the 22 questions (seven each for the pre-test and post-test) to formulate the test, with the focus being placed on the transition from perceived reality to mathematical models.

RESEARCH METHODOLOGY

Research subjects: The subjects were 58 first-year engineering students in a college, whose teachers had backgrounds in mathematics education with a rather comprehensive conceptual basis of mathematical modelling theories.

Research tool: This study aims at the development of the modelling competence of engineering students through mathematical modelling courses, where the construction of testing tools for evaluation of modelling competence is very important. The tools used include instructional activities for mathematical modelling, tests on modelling competence and semi-structured interviews; more details about the tools are given below:

- Mathematical modelling teaching and learning activities: the ultimate purpose of the instructional experiment is to cultivate student modelling competence through mathematical modelling classes; that is, to develop student capabilities in carrying out mathematical modelling tasks independently. The six modelling sub-competencies were designed in line with the six principles proposed by Lesh et al [15]. Taking the *Tracking problem* as an example, this teaching activity has been adopted from Yoon, Dreyfus and Thomas [16]. The problem statement presents the gradient graph of a tracking, asks students to develop a method for finding the distance-height graph of the original track, and to generalise their method so that it works for any gradient graph. By asking students to develop a method instead of merely providing their solution, the activity fulfils the *model construction principle*, and by asking them to generalise their method, it also fulfils the *model generalisation principle*. In mathematical terms, the problem amounts to creating a method for finding an anti-derivative of the given gradient graph, and the embedding of this mathematical task in the tracking context satisfies the *simple prototype principle*. The problem statement asks students to write their method in the form of a letter to clients who wish to determine whether the track is suitable for their purposes, thereby fulfilling the *model documentation principle*. Finally, students are instructed to use their method to draw the distance-height graph, which gives them a chance to test and revise their method, thereby satisfying the *self-assessment principle*.
- Testing competence in mathematical modelling: Table 1 briefly describes the situational questions in six categories and their corresponding modelling sub-competence (A stands for pre-test questions and B stands for post-test questions). Taking A2 and B4 as examples:

  A2: Consider the real world problem (do not try to solve it!):

  What is the best size for bicycle wheels?
  Which one of the following clarifying questions most addresses the smoothness of the ride?

  A. Are the wheels connected to the pedals by a chain?
  B. How tall is the rider?
  C. Has the bicycle got gears?

D. How high is the highest kerb that can be ridden up?
E. Does terrain matter?

B4: Consider the real world problem (do not try to solve it!):

The time required to evacuate an office block in an emergency needs to be known by the safety officer. There are conflicting needs of security, public access and ease of exit.

In a simple mathematical model, a single room is considered with people exiting that room in single file. Which one of the following options contains parameters, variables or constants, each of which should be included in the model?

A. Time elapsed after the alarm raised: Number of people evacuated by time *t*: Time of day at which the alarm sounded.
B. Number of people to be evacuated: Time elapsed after the alarm raised: Number of people evacuated by time *t*.
C. Number of people evacuated by time *t*: Time of day at which the emergency occurred: Width of the emergency exits.
D. Total time to evacuate everyone: Space between people leaving: Width of the emergency exits.
E. Speed of the line of people: Initial delay before the first person can leave: Amount of personal belongings carried out.

Table 1: Test on mathematical modelling competence.

Q1	A1/B1	placing a bus stop/tram route Which assumptions are the least important in formulating a simple mathematical model?
Q2	A2/B2	finding the best size for bicycle/pushchair wheels Which clarifying questions most address smoothness of the ride?
Q3	A3/B3	comparing single queue and multiple queue systems/comparing express and normal checkout Which problem should be used to complete the statement?
Q4	A4/B4	aircraft in an emergency/considering the time required to evacuate an office block Which parameters, variables, or constants should be included in the model?
Q5	A5/B5	linking the behaviour of a moving car/a growing sunflower Which of the given functions could describe behaviour that is appropriate?
Q6	A6/B6	an Olympic 100m sprinter's speed/an aircraft's speed Which graphs best represent the variation in the speed?
Q7	A7/B7	objects can pass beneath an electricity cable/objects can pass beneath a bridge Which objects can pass beneath the electricity cable/bridge?

- Semi-structured interview: to mend the deficiency in the *overall orientation* of the modelling test, semi-structured interviews were carried out during the analysis on the test papers, with the focus being placed on the ambiguities found in the student's writings. According to the result of analysis, 31 students were selected for interview, and the result of the interviews became the basis for the compilation of the coding and scoring system for mathematical modelling tests.

Data analysis: To rate the students' modelling competencies by grading their answers on the test papers, the researcher compiled a coded scoring system based on the answers to the test questions and the results of the semi-structured interviews, as shown in Table 2.

Table 2: Coded scoring sheet.

Level of answers	Score	Descriptions of the answering process
Level 3	3	Correct options are chosen; necessary modelling processes or the relationship between perceived reality and the mathematical world are considered and correctly implemented.
Level 2	2	The correct options are chosen; necessary modelling processes or the relationship between perceived reality and the mathematical world are considered, but knowledge of the perceived reality or mathematics is inadequate.
Level 1	1	Answers are not completely correct, but modelling processes or the relationship between perceived reality and the mathematical world is only partially considered, or knowledge of the perceived reality or mathematics is inadequate.
Level 0	0	1. Answers are correct or close to the correct option, despite uncertainty about the correctness of the options; no reasoning is provided. 2. Incorrect answers.

RESULT

The pre-test and post-test results for the students' modelling competencies are listed in Table 3. Observation of the results before (pre-test) and after (post-test) the implementation of instructional activities for modelling reveals that the average pre-test score is 0.96, and that of the post-test is 1.93. This suggests that before the instructional activity, the modelling activities were difficult for the first-year engineering students; but after the instructional activity, there was a 0.97 increase in the average score, demonstrating that the modelling instructional activity is evidently beneficial to the students in terms of their modelling competencies. Looking from an individual perspective, as to the students who were at Level 0, there were 12 to 19 students who chose the wrong answers to the problems in the pre-test, and most of them did not write down the reasoning for their choices.

During the interviews, most of them mentioned their difficulties with the pre-test, such as *couldn't understand the questions*, *have never dealt with similar questions before* and *guesswork was used for what I couldn't understand*. Clearly, there is room for improvement in the development of the students' modelling competencies at different levels. In the post-test, the number of items that were scored at 0 was substantially reduced, indicating that most students could choose the best or second best answer, and could provide a reason for their choice. Approximately 70% to 80% of students fell under Level 1 or Level 2 in both the pre- and post-test. Here differences also exist, in that there were more people that fell under Level 1 than Level 2 in the pre-test, whereas in the post-test, more people fell under Level 2 than Level 1. Individual comparisons of student performance would reveal the tendency that the students were moving from a lower level to a higher one. Likewise, the number of students at Level 3 has also increased significantly compared with that of the pre-test. Nevertheless, a majority of the students belonged to Level 1 and Level 2; in other words, they were far from being experts in modelling. Therefore, they were still unable to consider thoroughly the modelling process or the relationship between perceived reality and the mathematical world, or their knowledge of modelling it was inadequate.

Table 3: Statistical results of the pre-test and post-test of mathematical modelling competence.

N=58	0	1	2	3	M	SD
	n(%)	n(%)	n(%)	n(%)		
Structuring, simplifying situation (A1)	17(29.31)	23(39.66)	18(31.03)	0(0.00)	1.02	0.78
Structuring, simplifying situation (B1)	2(3.45)	15 (25.86)	28(48.28)	13(22.41)	1.9	0.79
Understanding ,interpreting context (A2)	27(46.55)	24(41.38)	7(12.07)	0(0.00)	0.7	0.7
Understanding ,interpreting context (B2)	3(5.17)	14(24.14)	28(48.28)	13(22.41)	1.9	0.8
Formulating mathematical statements (A3)	18(31.03)	26(44.83)	10(17.24)	4(6.90)	1	0.88
Formulating mathematical statements (B3)	2(3.45)	16 (27.59)	30(51.72)	10(17.24)	1.83	0.8
Assigning variables, parameters (A4)	17(29.31)	36(62.07)	4(6.90)	1(1.72)	0.8	0.6
Assigning variables, parameters (B4)	0(0.00)	11(18.96)	31(53.45)	16(27.59)	2.1	0.7
Selecting a model (A5)	17(29.31)	24(41.38)	14(24.14)	3(5.17)	1.1	0.9
Selecting a model (B5)	1(1.72)	14(24.14)	31(53.45)	12(20.69)	1.91	0.71
Using graphical representation (A6)	19(32.76)	28(48.28)	11(18.96)	0(0.00)	0.9	0.7
Using graphical representation (B6)	1(1.72)	17(29.31)	17(29.31)	23(39.66)	2.1	0.9
Interpreting solution in context (A7)	12(20.69)	31(53.45)	15(25.86)	0(0.00)	1.1	0.7
Interpreting solution in context (B7)	2(3.45)	19(32.76)	25(43.10)	12(20.69)	1.8	0.9

In terms of the students' individual modelling sub-competence, three categories that were insufficient during the pre-test - including *comprehension and interpretation of the situation*, *designating variables and parameters to the model* and *use of graphical representation* - showed the most significant improvement in the post-test. Regarding the aspect of *comprehension and interpretation of the situation*, the question in the pre-test was *What is the most suitable size for the wheel of toddler's stroller?*, while in the post-test it was *What is the most suitable wheel size for a bicycle?* Students were asked to choose the most suitable answer from the five options provided to satisfy the demands for comfort in the stroller and stability in the bicycle.

However, demands for comfort and stability induce qualitative thinking in students instead of quantitative thinking that resulted in difficulties in comprehending and interpreting the situation. John had the wrong choice of c) *Whether the seat is cushioned* during the pre-test, but had the correct option d) *If the height of the sidewalk makes it easier to get onto the bicycle* in the post-test. During the interview, he explained his choices: *Size is related to stability; it becomes unstable if the size is too small, but a reasonably larger size makes it more stable. Options a, b, and c have nothing to do with stability. I think the height of sidewalks is the most important consideration, for if the wheel is too big, my feet cannot touch the ground when I stop at traffic lights, despite the added stability; if the size of the wheel fits the sidewalk* *well, my feet can touch the ground after stepping off the pedals, saving me from getting on and off the bike repeatedly. So if I were to design the size of a bicycle wheel, I would measure the height of sidewalks first.*

The researcher then asked John to redo the pre-test question, and he chose *b) What is the distance between the front wheel and rear wheel of a toddler's stroller?* this time John responded: *I seem to have seen this question before; a and d are irrelevant to comfort, of course it would be more comfortable with a cushion, needless to say. A toddler's stroller is so small that the distance between the front and rear wheels does not matter much, even though it does play a part. I would choose the option* whether there is rubble on the sidewalk, or whether it is paved with wooden boards. *If there is rubble, I would design larger wheels to see what happens*. The researcher continued by asking John, *That is right, you did this before. Do you remember the answer you gave at that time?* John replied *No*, and was amazed when the research told him his original answer, saying, *Really? Did I actually choose this option? But this answer makes no sense!* A comparison of the contents of the pre-test and post-test would reveal, except for three students (those at Level 0), all had their capabilities markedly improved in terms of situational comprehension and interpretation, and clarification of the objective of the perceived reality.

As to the sub-competence of *assigning variables and parameters to the model*, the situation of concern was described as *Holding an emergency fire drill to assess the time required for evacuation*, and the students were required to pick out the parameters, variables, or limitations necessary for constructing the mathematical model. Taking Tom as an example, in the pre-test he chose the second best option: *total evacuation time of the students, distance to maintain between students while evacuating, and the width of the library doors*, without providing the reason for his choice; therefore, no score was given.

In the post-test, he chose the best option, *the total number of people to evacuate, time elapsed after the alarm went off, the number of people evacuated by time t,* and he gave his reasoning in the interview: *the total time for evacuation is related to the total number of people to be evacuated - the more people there are, the longer the amount of time that is needed. The time elapsed after the alarm went off is the total time. As to the number of people evacuated at time t, it varies if time t is different, which is very important and certainly should be considered. What about the width of the emergency exit?* asked the researcher. *The width must be considered too, of course, but it is already given as a condition of the question that the building has set up space to allow a single-person queue for exit; granted that the width is fixed, it should be considered as a constant rather than a variable setting,* answered Tom, with obvious mastery of the situational knowledge.

The researcher continued his questions: *How about the speed of evacuation?* Tom answered, *The speed of evacuation is certainly related to the total time, thus it should be set as a parameter to examine the relationship with the total time*. In fact, the students had a hard time assigning variables and parameters to the model during the modelling activities; for them, this was a brand new mathematical experience. Observations of the first three mathematical modelling activities reveal that most students could not distinguish between parameter, variable and constant. Therefore, apart from enhancing modelling sub-competence in terms of *assigning variables and parameters to the model*, understanding the meaning of, and difference between parameters, variables and constants is an extra benefit of the mathematical modelling instructional activities.

CONCLUSIONS

This study shows that the modelling competence of the students before the modelling course was fairly weak, and this echoes with the opinions held by Haines et al [12]. Among the many aspects of the weakness, most students were particularly deficient in terms of the modelling sub-competencies of *comprehension and interpretation of situations*, *assigning variables and parameters to the model* and *graphical representation*.

After taking the modelling classes, the modelling capabilities of the students had clearly been strengthened, but they were still far from being experts in modelling. This also shows that the mathematical modelling capability is a sophisticated expert capability, which takes a long time and lots of practice to attain. The author suggested that future research can extend modelling courses to the curriculum for sophomore engineering students, offering longer learning time to foster their mathematical modelling competence.

This study focuses on real life scenarios, where students can reflect their understanding of mathematics, mathematisation, mathematical systems analysis and interpreting the results. Above all, students are presented with an opportunity to learn mathematics in a different way. Mathematical modelling should be considered pivotal to university mathematics or engineering education as it connects the real world with the mathematical world, and can stimulate the learning process and help students develop key concepts and notions of mathematics.

However, it is very important to strike a balance between the development of mathematical modelling competence, and the comprehension of mathematical concepts through instructional activities for mathematical modelling.

REFERENCES

1. Freudenthal, H., Why to teach mathematics so as to be useful? *Educational Studies in Mathematics,* 1, **1/2**, 3-8 (1968).
2. Gravemeijer, K. and Doorman, M., Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics,* 39, 111-129 (1999).
3. Lesh. R. and Doerr, H.M., *Beyond Constructivism: Models and Modelling Perspectives on Mathematics Problem Solving, Learning, and Teaching.* Lawrence Erlbaum Associates, Inc. (2003).
4. Burkhardt, H., Modelling in mathematics classrooms: reflections on past developments and the future. *Zentralblatt für Didaktik der Mathematik*, 38, **2**, 178-195 (2006).
5. Blomhoj, M., *Mathematical Modelling – A Theory for Practice*. In: Clarke, B., Clarke, D.M., Lambdin, D.V. and Lester, F.K. (Eds), International perspectives on learning and teaching mathematics. Goteborg University: National Center for Mathematics Education, 145-160 (2004).
6. Maaß, K., What are modelling competencies? *Zentralblatt für Didaktik der Mathematik,* 38, **2**, 113-142 (2006).
7. Blum, W. and Niss, M., Applied mathematical problem solving, modelling, applications, and links to other subjects - state, trends and issues in mathematics instruction. *Educational Studies in Mathematics*, 22, 37-68 (1991).
8. Blomhoj, M. and Jensen, T.H., Developing mathematical modelling competence: Conceptual clarification and educational planning. *Teaching Mathematics and its Applications,* 22, **3**, 123-139 (2003).
9. Rowland D.R. and Jovanoski, Z., Student interpretations of the terms in first-order ordinary differential equations in modelling contexts, *Inter. J. of Mathematical Educ. in Science and Technol.,* 35, **4**, 503-516 (2004).
10. Chaachoua, H. and Saglam, A., Modelling by differential equations, *Teaching Mathematics and its Applications,* 25, **1**, 15-22 (2006).
11. Rowland, D.R., Student difficulties with units in differential equations in modelling contexts, *Inter. J. of Mathematical Educ. in Science and Technol.,* 37, **5**, 553-558 (2006).
12. Haines, C.R., Crouch, R.M. and Davis, J., Mathematical Modelling Skills: A Research Instrument. University of Hertfordshire. Department of Mathematics Technical Report No. 55. Hatfield: University of Hertfordshire (2000).
13. Houston, K. and Neill, N., *Assessing Modelling Skills*. In: Lamon, S.J., Parker, W.A. and Houston, S.K. (Eds), Mathematical Modelling: A Way of Life ICTMA 11. Chichester: Horwood Publishing, 155-164 (2003).
14. Izard, J., Haines, C., Crouch, R., Houston, K. and Neill, N., *Assessing the Impact of Teachings Mathematical Modelling: Some Implications*. In: Lamon, S.J., Parker, W.A. and Houston, S.K. (Eds), Mathematical Modelling: A Way of Life ICTMA 11. Chichester: Horwood Publishing 165-177 (2003).
15. Lesh, R., Hoover, M., Hole, B., Kelly, A. and Post, T., *Principles for Developing Thought-Revealing Activities for Students and Teachers*. In: Kelly, A.E. and Lesh, R. (Eds), Handbook of Research Design in Mathematics and Science Education. Mahwah, New Jersey: Lawrence Erlbaum, 591-646 (2000).
16. Yoon, C., Dreyfus, T. and Thomas, M.O.J., How high is the tramping track? Mathematising and applying in a calculus model-eliciting activity. *Mathematics Educ. Research J.,* 22, **2**, 141-157 (2010).